References

[1] J. R. Whinnery, H. W. Jamieson, and T. E. Robbins, "Coaxial-line discontinuities," *Proc. IRE.*, vol. 32, pp. 695–709, Nov. 1944.

[2] E. W. Risley, Jr., "Discontinuity capacitance of a coaxial line terminated in a circular waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 86–92, Feb. 1969.

[3] E. W. Risley, Jr., "Discontinuity capacitance of a coaxial line terminated in a circular waveguide: Part II—Lower bound solution," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 564–566, Aug. 1973.

[4] R. Safavi-Naini and R. H. MacPhie, "On solving waveguide junction scattering problems by the conservation of complex power technique," *IEEE Trans. Microwave Theory Tech.*, vol MTT-29, pp. 337–343, Apr. 1981.

[5] R. Safavi-Naini and R. H. MacPhie, "Scattering at rectangular-to-rectangular waveguide junctions," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 2060–2063, Nov. 1982.

[6] R. R. Mansour and R. H. MacPhie, "A unified hybrid-mode analysis for planar transmission lines with multilayer isotropic/anisotropic substrates," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 1382–1391, Dec. 1987.

[7] R. Mittra and S. W. Lee, *Analytical Techniques in the Theory of Guided Waves.* New York: Macmillan, 1971.

[8] G. A. Deschamps, "Determination of the reflection coefficients and insertion loss of a waveguide junction," *J. Appl. Phys.*, vol. 24, pp. 1046–1050, Aug. 1953.

[9] T. Do-Nhat and R. H. MacPhie, "On the effect of gap width on the admittance of solid circular cylindrical dipoles," *IEEE Trans. Antennas Propagat*, vol. 37, pp. 1545–1553, Dec. 1989.

An Experimental Technique for *In Vivo* Permittivity Measurement of Materials at Microwave Frequencies

KATIE F. STAEBELL AND DEVENDRA MISRA, MEMBER, IEEE

***Abstract*—This paper involves the formulation of a new procedure to be used in making *in vivo* dielectric measurements with an open-ended coaxial line probe. The theory behind the technique is discussed along with a correction method to account for the system imperfections. Experimental results are compared with the corresponding data available in the literature. The limitations of this technique are considered.**

I. Introduction

In order to understand the interaction of an electromagnetic field with a material medium, it is important to accurately know its complex permittivity. This information is also desired in many areas of science and engineering, including process control in industries and diagnostic and therapeutic application of microwaves in biomedical engineering. Classical methods of measuring the permittivity of a material require special preparation of a sample that is placed inside a waveguide or cavity and, hence, are not suited for many modern applications. For example, in the case of biological materials, *in vivo* properties are desired. An open-ended coaxial sensor has attracted several researchers because of its applicability in nondestructive measurement over a broad frequency band. However, the relation between the measured reflection coefficient and complex permittivity of the material is not simple. Several attempts have been made to devise a practical way of determining the permittivity from the measured data [1]. Recently, Marsland and Evans reported a bilinear transformation scheme to account for the imperfection in the measuring system in conjunction with an equivalent circuit model for the coaxial opening [2]. However, this technique is restricted at high frequencies by the inadequate circuit model for the probe.

The present approach is based on a quasi-static analysis of the coaxial sensor to formulate a relatively more accurate equivalent model [3], [4]. The experimental results determined by this method are compared with the corresponding values available in the literature over the frequency range of 1 to 20 GHz. The limitations of this technique are also considered.

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The authors are with the Electromagnetic Communication Research Laboratory, Department of Electrical Engineering, University of Wisconsin-Milwaukee, P.O. Box 784, Milwaukee, WI 53201.

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II. Theoretical Background

A stationary formula for the aperture admittance of an open-ended coaxial line terminated by a semi-infinite medium on a ground plane is given in [3]. Under a quasi-static approximation, this expression reduces to

$$Y_L \simeq j\frac{2\omega I_1}{[\ln(b/a)]^2}\epsilon^* - j\frac{\omega^3\mu_0 I_2}{[\ln(b/a)]^2}\epsilon^{*2} + \frac{\pi\omega^4\mu_0^{3/2}}{12}\left[\frac{b^2-a^2}{\ln(b/a)}\right]^2\epsilon^{*5/2} \quad (1)$$

where a and b are the inner and outer radii of coaxial aperture, respectively; μ_0 is the permeability of free space; ϵ^* is the complex permittivity of the semi-infinite medium; ω is the angular frequency of electromagnetic fields; and I_1 and I_2 are two triple integrals dependent on the radii but constant otherwise [3]. The medium under consideration is assumed to be linear, isotropic, homogeneous, and nonmagnetic. Other details of this formulation are available in [3] and, therefore, are omitted for the sake of brevity.

The first term of (1) represents a capacitance; the third one, radiation conductance, is used in an equivalent circuit model by several researchers [1], [2]. The second term of this equation represents a frequency-dependent capacitance, which is new and probably responsible for restrictions at high frequencies in the work reported by Marsland and Evans [2]. It is to be noted that this term is important in comparison to the radiation conductance (third term) because of its contribution even around 1 GHz.

III. System Calibration

In formulating (1), an infinite conducting flange is assumed over the coaxial aperture. However, it is not used in practice because it is inconvenient. Also, small discontinuities between the aperture and the reflection meter (due to connectors, etc.) cannot be avoided. In order to account for these imperfections, we consider an equivalent two-port network connected between the meter and the coaxial opening. The actual admittance of the aperture terminated by a sample is evaluated from the measured reflection coefficient after calibrating the system with three standard materials as follows [2], [4]:

$$\frac{Y_s - Y_1}{Y_s - Y_2}\cdot\frac{Y_3 - Y_2}{Y_1 - Y_3} = \frac{\delta_{s1}\delta_{32}}{\delta_{s2}\delta_{13}} \quad (2)$$

where Y_s is the desired aperture admittance terminated by the

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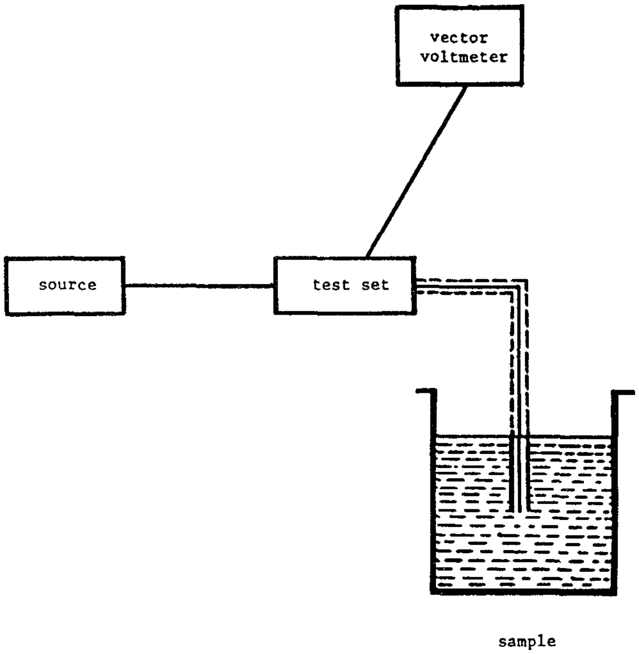

Fig. 1. Experimental setup for measuring reflection coefficients.

sample material; $Y_{1,2,3}$ are aperture admittances with standards, respectively; and $\delta_{ij} = \Gamma_i - \Gamma_j$ with Γ_n representing the measured reflection coefficient for the nth material.

The right-hand side of (2) is determined from the measured data while $Y_{1,2,3}$ are calculated from (1) since the values of ϵ^* for the standards are known. It is to be noted at this point that the radii of the annular opening and the values of integrals I_1 and I_2 are needed to calculate the Y's, which is not very convenient in practice. This can be avoided by using a fourth standard and noting the bilinear transformation characteristic of the admittance as follows.

From (1) one can write

$$y_L = \epsilon_r^* + \xi \epsilon_r^{*2} + \xi_1 \epsilon_r^{*2.5} \tag{3}$$

where ϵ_r^* is the complex relative permittivity of the material, ξ and ξ_1 are constants dependent on the frequency and the dimension of the aperture, and y_L represents a transformed parameter of aperture admittance Y_L.

Generally, the radiation from the coaxial aperture can be neglected at lower microwave frequencies. Therefore, (3) can be approximated as follows:

$$y_L \simeq \epsilon_r^* + \xi \epsilon_r^{*2}. \tag{4}$$

Equations (2) and (4) can be used to determine the aperture parameter y_s which, in turn, can be used to calculate the complex permittivity of the sample as summarized below, assuming that the third standard used is a short circuit:

$$y_s = (y_1 + \Delta' y_2)/(1 + \Delta') \tag{5}$$

where $\Delta' = \delta_{s1}\delta_{32}/\delta_{s2}\delta_{13}$. The values of $y_{1,2}$ are calculated from (4) for given complex relative permittivities of the standards 1 and 2, respectively, at the operating frequency. The unknown constant ξ in (4) is calculated from the measured reflection coefficients and known complex permittivities for standards 1, 2, and 4 as follows:

$$\xi = \frac{(1+\Delta)\epsilon_4^* - \epsilon_1^* - \Delta\epsilon_2^*}{\epsilon_1^{*2} + \Delta\epsilon_2^{*2} - (1+\Delta)\epsilon_4^{*2}} \tag{6}$$

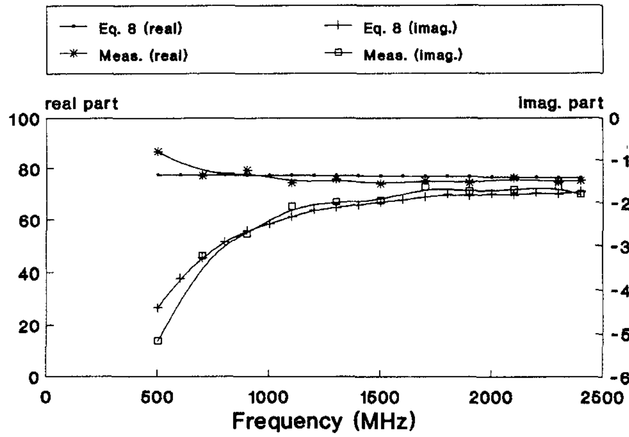

Fig. 2 Complex relative permittivity of 0.1N saline solution at 23°C.

and

$$\Delta = \delta_{41}\delta_{32}/\delta_{42}\delta_{13}. \tag{7}$$

The quantities δ_{ij} were defined earlier.

IV. The Measurement Procedure

The experimental setup for measuring different reflection coefficients, Γ_i, is shown in Fig. 1. The desired microwave signal was derived from an HP-8620C sweep oscillator equipped with an HP-86222A RF plug-in (frequency range 0.01 to 2.4 GHz). The incident and reflected signal outputs of an HP-85044A transmission/reflection test set (frequency range 300 kHz to 3 GHz) were coupled to an HP-8508A vector voltmeter (frequency range 100 kHz to 1 GHz), which displays the magnitude and phase of the reflection coefficient.

The model developed in Section II of this paper is based on the presence of an infinite sample. Therefore, line size should be selected to ensure that fringing and radiation fields are contained within the material [10]. This cable is inserted into the reflection–transmission test set. The system generator is used for choosing the desired frequency. The vector voltmeter provides the readings for the magnitude and phase of the complex reflection coefficient. This instrument, upon the manufacturer's calibration, has an uncertainty of ± 0.5 dB in magnitude and $\pm 6°$ in phase readings over the operating range of 500 MHz to 1 GHz. According to the meter's specifications, the instrument can lock to a source signal up to 2 GHz. During experimentation, source signals up to 2.4 GHz were used in obtaining data, indicating that the instrument has the ability to lock to source signals slightly beyond its specification. The error associated with the frequencies beyond the manufacturer's calibration range (100 kHz–1 GHz) is minimized by the calibration procedure outlined in the previous section. System discontinuities occurring from the tip of the probe backward up to the meter also cancel out upon calibration. After using water, an open circuit, a short circuit, and methanol as standards 1, 2, 3, and 4, respectively, we measured the complex permittivities of several materials. These results are found to be in good agreement with the corresponding data available in the literature. However, only a few results are being presented in the following section, for brevity.

V. Results and Discussion

Following the procedure presented in the preceding sections, the complex relative permittivities of several samples were determined and compared with the corresponding data available in

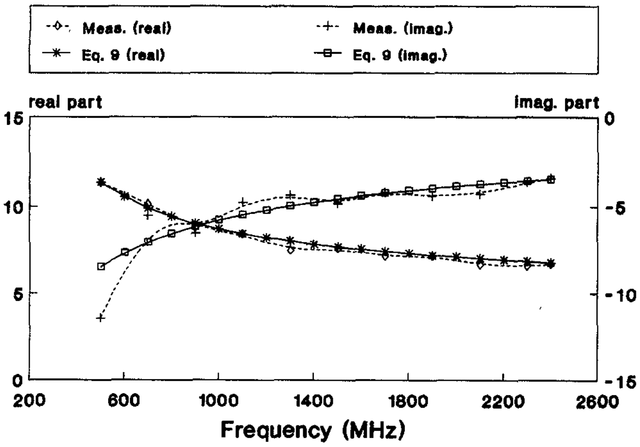

Fig. 3. Complex relative permittivity of glycerol at 23°C.

TABLE I
COMPLEX PERMITTIVITIES OF TWO DIOXANE–WATER SOLUTIONS

Solution	Frequency GHz	Present Method	From Reference [9] Lumped Parameter Model	From Reference [9] Integral Equation	From Reference [9] Literature
80% dioxane and 20% water	1	11 - j1	11 - j1	11 - j1	11 - j1
	3	10 - j3	10 - j2	10 - j3	10 - j3
	10	6 - j3	7 - j1	6 - j3	7 - j3
	18	5 - j2	5 + j0.5	5 - j3	5 - j3
60% dioxane and 40% water	1	25 - j3	26 - j3	26 - j3	26 - j3
	3	22 - j7	22 - j6	23 - j7	23 - j7
	10	12 - j10	15 - j6	10 - j10	14 - j10
	18	8 - j7	10 - j3	8 - j7	9 - j9

the literature. Since (4) is quadratic in ϵ_r^*, mathematically two solutions of complex permittivity are found. However, the linear approximation of this equation (i.e., $\xi = 0$) can be used at this point to determine the correct permittivity from the two mathematical solutions.

Fig. 2 depicts the complex relative permittivity of a 0.1N saline solution determined by the present technique at room temperature. These results are compared with the available data after the desired interpolation [5] and formulating the following dispersion equation:

$$\epsilon_r^* = 4.9 + \frac{72.72}{1 + j54.76 \cdot 10^{-12} \cdot f} - j\frac{21.06 \cdot 10^9}{f}. \tag{8}$$

At 500 MHz, the measured complex permittivity of $87 - j52$ is far from the value $78 - j44$ obtained from (8). This discrepancy is attributed to the system limitations as explained in [2]. When the signal frequency was changed to 700 MHz, the measured value of $77 - j32$ was found, which is in excellent agreement with the $78 - j33$ obtained from (8). Similar results may be seen in Fig. 2 at other higher frequencies. The complex permittivity of glycerol determined by the present method at 23° is shown in Fig. 3. These results are very close to the values calculated by the following equation [6]–[8]:

$$\epsilon_r^* = 4.18 + \frac{38.32}{\left[1 + j\omega \cdot 2.49 \cdot 10^{-9}\right]^{0.6}} \tag{9}$$

where ω is the angular frequency of the microwave signal and ϵ_r^* is the complex relative permittivity of glycerol.

Finally, as a comparison of the present technique with the lumped parameter model approach as well as with a mathematically sound integral equation method [9], the results of permittivities of two different dioxane–water solutions are listed in Table I, which also includes the corresponding values available in the literature. These results show good agreement, with the exception of the lumped parameter technique, which deviates at higher frequencies.

VI. CONCLUSION

With the proper choice of calibration standards and the use of the formulated admittance model, the complex permittivities of various unknowns can be determined over a wide range of frequencies. This experimental technique uses a system consisting of a coaxial line connected to a reflection transmission test set, a frequency generator, and a vector voltmeter or a network analyzer. The coaxial line is terminated with four standards, calibrating the meter for further measurement. Reflection coefficient data are taken for the four standards as well as an unknown. The permittivity of this sample is determined using the experimental data and a mathematical model for the aperature admittance.

Due to the assumption of an infinite sample, it is important that the fringing and radiation fields be contained within the sample under test. The cable size should be selected accordingly. The standards used for calibration should be chosen so their dielectric constants span a wide range of values. This type of selection proved to generate more accurate results, especially at high frequencies.

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REFERENCES

[1] M. A. Stuchly and S. S. Stuchly, "Coaxial line reflection method for measuring dielectric properties of biological substances at radio and microwave frequencies—A review," *IEEE Trans. Instrum. Meas.*, vol. IM-29, no. 3, pp. 176–183, Sept. 1980.

[2] T. P. Marsland and S. Evans, "Dielectric measurements with an open-ended coaxial probe," *Proc. Inst. Elec. Eng.*, vol. 134, pt. H, no. 4, pp. 341–349, Aug. 1987.

[3] D. K. Misra, "A quasi-static analysis of open-ended coaxial lines," *IEEE Trans Microwave Theory Tech.*, vol. MTT-35, pp. 925–928, Oct. 1987.

[4] M. Chabbra, S. Q. Fan, and D. Misra, "A study on the open-ended coaxial line method for measuring the permittivity of materials at microwave frequencies," in *Proc. IEEE IMTC/89 Conf.* (Washington, D.C.). April 25–27, 1989 pp. 541–545.

[5] J. A. Saxton and J. A. Lane, "Electrical properties of sea water—Reflection and attenuation characteristics at V.H.F., "*Wireless Engr.*, pp 269–275, Oct. 1952.

[6] F. Buckley and A. A. Maryott, *Tables of Dielectric Dispersion Data for Pure Liquids and Dilute Solutions*, National Bureau of Standards Circular 598, Nov. 1958.

[7] D. W. Davidson and R. H. Cole, "Dielectric relaxation in glycerine," *J. Chem. Phys.*, vol. 18, p. 1417, 1950.

[8] S. O. Morgan, "Two types of dielectric polarization," *Trans. Amer. Electrochem. Soc.*, vol. 65, pp. 109–118, 1934.

[9] D. Misra, M. Chabbra, B. R. Epstein, M Mirstznik, and K. R. Foster, "Noninvasive electrical characterization of materials at microwave frequencies using an open-ended coaxial line: Test of an improved calibration technique," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 8–14, Jan. 1990.

[10] L. S. Anderson, G. B. Gajda, and S. S Stuchly, "Analysis of an open-ended coaxial line sensor in layered dielectrics," *IEEE Trans. Instrum. Meas.*, vol. IM-35, pp. 13–18, Mar. 1986.